Luminosity variation in (de)focusing microlensing

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Abstract

The luminosity variation of a stellar source due to the gravitational microlensing effect can be considered also if the light rays are defocused (instead of focused) toward the observer. In this case, we should detect a gap instead of a peak in the light curve of the source. Actually, we describe how the phenomenon depends on the relative position of source and lens with respect to the observer: if the lens is between, we have focusing, if the lens is behind, we have defocusing. It is shown that the number of events with predicted gaps is equal to the number of events with peaks in the light curves.

1 Introduction

Microlensing is a specific application of gravitational lensing which is mainly used to search for the *Massive Astrophysical Compact Halo Objects* (MACHOs) [1], which are today considered as the most probable constituents of the dark halo of our Galaxy [2], [3]; however other possibilities are today explored. In [4],[5],[6], microlensing by cold dark matter particles and noncompact objects is considered.

The term "microlensing" is used since the angular separation of the two images, usually produced by a point–like mass lens, is too small to be resolved ($\sim 10^{-6}$ arcsec). However, even if it is not possible to detect multiple images, the magnification can still be seen when the lens and the source move relatively to each other: this motion gives rise to a lensing–induced time variability of the source luminosity [7]. The effect was firstly observed for quasars [8],[9], so that we have to distinguish *galactic* and *extragalactic* microlensing. In the first case, the light sources are stars and the angular separations involved are $\sim 10^{-3}$arcsec, in the second case, the sources are quasars and the angular separations involved are $\sim 10^{-6}$arcsec. The term "microlensing" is used in both cases. The principle on which the phenomenon is based is the following. If the closest approach between a point mass lens and a source is equal or less than the Einstein angular radius θ_E, the peak magnification in lensing–induced light curve corresponds to a brightness enhancement which can be easily detected with the today facilities [1]. The Einstein angular radius θ_E is a feature of the system lens–source–observer which furnishes the natural angular scale to describe the lensing geometry. Starting from the gravitational lens equation [10], it is given by

$$\theta_E = \sqrt{\frac{4GM(\leq r_E)D_{ls}}{c^2 D_{ol}D_{os}}} \,, \tag{1}$$

where D_{ls}, D_{ol}, D_{os} are respectively the distances lens–source, lens–observer, and source–observer. The angular distance θ_E corresponds to the effective distance $r_E = \theta_E D_{ol}$. The symbol $M(\leq r_E)$ means that the mass of the lens has to be contained inside a sphere whose radius is the Einstein one. For multiple imaging, θ_E gives the typical angular separation among the single images; for axisymmetric lens–source–observer systems, it gives the aperture of a circular bright image, called *Einstein ring*. Sources which are closer than θ_E to the optical axis experience strong lensing effect and are hardly magnified, sources which are located well outside of the Einstein ring are not very much magnified. In other words, for a lot of lens models, the Einstein ring represents the boundary between the zones where sources are strongly magnified or multiply–imaged and those where they are softly magnified or singly–magnified [10].

In order to detect microlensing, (and then MACHOs) the first proposal [1] was to monitor millions of stars in the *Large Magellanic Cloud* (LMC), or in the bulge of Galaxy in order to look for such magnifications. If enough events are detected, it should be possible to map the distribution of (dark) mass objects in the halo of Galaxy or between the Solar System and the bulge of Galaxy. Due to the distances involved, both approaches

can be used for "galactic microlensing" [1],[11]. The expected time scale for microlensing–induced luminosity variations is given in terms of the typical angular scale θ_E, the relative velocity v between source and lens, and the distance of the observer to the lens D_{ol}, that is $\Delta t = (D_{ol}\theta_E)/v$. If light curves are sampled with time intervals between the hour and the year, the mass range of MACHOs is $10^{-6} \div 10^2 M_\odot$, that is from planets to very massive stars (or black holes). These numbers are in agreement with theoretical constraints [3],[10]. The chance of seeing microlensing events depends on the *optical depth*, which is the probability that at any instant a given source is within the angle θ_E of a lens, as we shall see below.

Several groups are searching for MACHOs [11],[12], [13],[14] but so far, few experimental data (about 100 events) can be considered statistically relevant in order to allow to draw conclusions on the physical properties of MACHOs like their mass.

An important point has to be discussed. Until now, microlensing is considered for lenses which focus light rays toward the observer. On the other hand, in optics, there exists the opposite effect if the refraction index of media is appropriately chosen and if the relative positions of the source and the lens is changed with respect to the observer. That is, we can ask the question whether there exist or not distributions of matter producing gravitational fields which deflect the light rays in a manner which mimics defocusing lenses of usual optics. The wish to introduce and to study the notion of *defocusing gravitational lens* is mainly motivated by the hypothesis that the microlensing events with luminosity peak may be accompanied by the existence of events with valley in the luminosity curve. This inverse phenomenon can be easily understood by the following considerations. In the "standard" studied situation, a MACHO is between the source and the observer and the emitted rays are slightly curved in the direction of the observer and such a fact produces the a luminosity magnification. The opposite situation is statistically as probable as the previous one when a MACHO is located behind the source with respect to the observer. Then, the source rays are slightly curved out of the observer direction which detects a gap in luminosity. We will discuss precisely this situation using the equations for geodesics (along which light rays move) in a generic Schwarzschild gravitational field.

In this paper, we discuss the (de)focusing microlensing considering a simple model in which a MACHO moves with respect to the source and the observer. However, the discussion can be generalized in a statitistical way by considering several sources and lenses.

2   Luminosity variation induced by a point mass lens

If θ_s is the angular size of the source and the condition $\theta_s \leq \theta_E$ holds, the magnification due to the microlensing effect must be $\mu \geq 1.34$, (see, for example, [10],[15]). A magnification $\mu \sim 1.34$ corresponds to a magnitude enhancement of $\Delta m \sim 0.32$. In other words, we can say that when the true position of a light source lies inside the Einstein ring, the total magnification of the two images that it yields amounts to $\mu \geq 1.34$. This means

that the angular cross section for having significant microlensing effects (*i.e.* $\mu \sim 1.34$ and $\Delta m \sim 0.32$), is equal to $\pi\theta_E^2$. Such a cross section, from (1), is proportional to the mass M of the deflector and to the ratio of the distances involved. Such considerations allow to calculate the *optical depth*.

Let us consider the case of randomly distributed point–mass lenses: it is possible to estimate the frequency of significant gravitational lensing events from the observations of distant compact sources, that is we are considering optical systems where the involved angular sizes are much smaller than θ_E. In this situation, the magnification of a compact source is equal or greater than 1.34 (since $\theta_s < \theta_E$) and the probability P to have a significant microlensing event for a randomly located compact source at a distance D_{os} is given by

$$P = \frac{\pi\theta_E^2}{4\pi} = \left(\frac{D_{ls}}{D_{os}D_{ol}}\right)\left(\frac{GM}{c^2}\right) . \tag{2}$$

Such a probability is linear in the mass M of deflector so that it holds also when several point–mass lenses are acting. Assuming a constant density for the lens(es) and a static background (this last assumption surely holds for galactic distances), averaging opportunely on the distances D_{ls}, D_{ol}, D_{os}, the probability (2) can be interpreted as the optical depth τ for lensing [15],[16],[17]:

$$P = \tau = -\left(\frac{D_{ls}}{D_{os}}\right)\frac{U}{c^2} , \tag{3}$$

where $U = -GM/D_{ol}$, is the Newton potential due to the lens as measured by the observer. In other words, τ corresponds to the fraction of sky covered by the Einstein ring. Due to the fact that the deflecting masses change the path of light rays, the observer will detect different luminosities for a given source when the deflector is present and when it is not present: then, the optical depth is related with such a relative luminosity change as we shall see below.

Before discussing how to realize (de)focusing, we have to consider the motion of light ray paths in a gravitational field in order to obtain the luminosity variations due to the presence of point mass lenses. We have to take into account the geometric optic approximation since we are assuming that light propagates as rays.

As it is known [10], a gravitational field has the same effect of a medium in which light rays propagates. For weak gravitational fields, the metric tensor components $g_{\mu\nu}$ can be expressed in terms of Newton gravitational potential Φ. The refraction index n, in this case, is related to the gravitational potential $\Phi(r)$ produced by some matter distribution, that is $n = 1 - 2\Phi(r)/c^2$. If the rays pass near a spherical body of mass M, they will undergo the action of a Schwarzschild gravitational field described by the metric element

$$ds^2 = \left(1 - \frac{R_s}{r}\right)c^2t^2 - \frac{dr^2}{\left(1 - \frac{R_s}{r}\right)} - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) , \tag{4}$$

where $R_s = 2MG/c^2$ is the Schwarzschild radius. The light ray trajectories passing near the deflecting body can be easily found by solving the problem of motion connected with

(4). If the condition $r \gg R_s$ holds (that is the light ray passes well outside of the surface where the metric becomes singular), the trajectory is

$$r = r_0 \left\{ \cos(\phi - \phi_0) + \frac{R_s}{2r_0} \left[2 - \cos^2(\phi - \phi_0) \right] \right\}^{-1} , \tag{5}$$

which is nothing else but a straight line corrected by a hyperbolic–like term in polar coordinates [18]. The parameters r_0 and ϕ_0 are the initial data of the problem; r_0 is the distance of the line from the origin of coordinates, ϕ_0 is a given angle which tells us how much the line is tilted with respect to the polar axis. The deflecting mass is set at the origin of reference frame. The amount of deviation from the rectilinear behaviour depends on the ratio R_s/r_0, that is on the mass M of the gravitational source and on the parameter r_0. In Cartesian coordinates Eq.(5) becomes

$$r_0 = Ax + By + \left(\frac{R_s}{r_0} \right) \sqrt{x^2 + y^2} - \left(\frac{R_s}{2r_0} \right) \frac{(Ax + By)^2}{\sqrt{x^2 + y^2}} , \tag{6}$$

where $A = \cos \phi_0$ and $B = \sin \phi_0$.

Let us consider now the limit $r \to \infty$. Eq.(5) becomes an algebraic equation for $\cos(\phi - \phi_0)$ from which we get (being $r_0 \gg R_s$)

$$\cos(\phi - \phi_0) \simeq -\frac{2MG}{c^2 r_0} \tag{7}$$

which indicates how the gravitational field ($M \neq 0$) deviates the rays from the straight line direction. If $M = 0$ or $r_0 \to \infty$ (that is in absence of gravitational field or when r_0 is very large), we have $\cos(\phi - \phi_0) = 0$, that is $\phi - \phi_0 = \pm\frac{\pi}{2}$. If the gravitational field is weak, in the limit $r \to \infty$, we have $\phi - \phi_0 = \pm(\delta + \pi/2)$, from which, by substituting into (5), we get $\sin \delta \simeq \delta = R_s/r_0$ being δ small. The total amount of ray deviation gives the standard result $2\delta \simeq 4MG/(c^2 r_0)$, which is the deflection angle due to a point mass acting as a gravitational lens.

Now, taking in mind such a results, we want to obtain the general formula describing the variation of luminosity of a radiation source in the sky induced by a gravitational microlensing effect. We will show that such a variation is due to the change of direction of light rays (geodesics) which move in a given nonstationary matter distribution. In other words, we are supposing that a given background metric $g_{\mu\nu}^{(1)}$ is modified by a passing heavy body (a MACHO) which locally perturbs it so that we have to consider a new metric $g_{\mu\nu}^{(2)}$. The effect of such a background change is a deviation in the direction of geodesics which gives a bundle of hyperbolic–like curves (instead of the unperturbed bundle of straight lines). Two cases are possible: the observable variation of source luminosity is due to a microlensing focusing effect or to a microlensing defocusing effect. In the first case, a MACHO is between the source and the observer producing focusing, that is, at a certain moment, the alignment (I)

$$\text{source} \quad \longrightarrow \quad \text{lens} \quad \longrightarrow \quad \text{observer} ,$$

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is realized; in the second case, a MACHO is behind the source and light rays are defocused toward the observer, that is, at a certain moment the alignment (II)

$$\text{lens} \quad \longrightarrow \quad \text{source} \quad \longrightarrow \quad \text{observer}\,,$$

is realized. In the first case, the observer detects an increasing luminosity, in the second case, he detects a decreasing one. The problem can be easily formulated by a geometric model in which, given a reference frame, we assign the position of the light source and the position of the detector (a telescope) in a background metric $g_{\mu\nu}^{(1)}$. Then we calculate the geodesics which give the light–ray paths. Furtherly, considering a MACHO passing between the source and the observer or behind the source (with respect to the observer), the metric becomes locally $g_{\mu\nu}^{(2)}$ and the geodesics will change giving focusing or defocusing of light rays. Let us start by choosing a system of Cartesian coordinates. We put the source in $(x_S,\, y_S) = \{-a,\, 0\}$ in the case of configuration (I) and $(x_S,\, y_S) = \{a,\, 0\}$ in the case of configuration (II). The telescope is in $(x_T,\, y_T) = (R,\, h)$ and we are assuming that the MACHO passes in the origin $(0,\, 0)$. At the beginning, it is not present and the metric is $g_{\mu\nu}^{(1)}$. There exists a unique light ray (a unique geodesic) which intersects the source and the upper limit of the telescope aperture.

Let us now suppose that, due to a redistribution of matter, the metric becomes $g_{\mu\nu}^{(2)}$, (the simplest case is to consider a passing MACHO). This event modifies the structure of geodesic bundle from the source to the observer. Schematically, we have a different bundle geodesics between the source and the upper limit of the aperture of the telescope. The rays which reach the upper limit of the telescope in the metric $g_{\mu\nu}^{(1)}$ are emitted at the angle α_1, while they are emitted at the angle α_2 in the metric $g_{\mu\nu}^{(2)}$. In the first case, each geodesic is given by a function $y_1(x)$ in Cartesian coordinates; in the second one by a function $y_2(x)$. The angles α_1 and α_2 are given by the derivatives

$$\tan\alpha_1 = \left.\frac{dy_1}{dx}\right|_{(\mp a;0)}\,, \qquad \tan\alpha_2 = \left.\frac{dy_2}{dx}\right|_{(\mp a;0)}\,, \qquad (8)$$

calculated in the coordinates of the source. The variation of luminosity is related to the variation of the direction of geodesics, that is to the change of the number of light rays which reach the telescope, so that the general formula for the relative change of luminosities in both cases is

$$\frac{\Delta L}{L} = \pm\left[\left(\frac{dy_2(x)}{dx} - \frac{dy_1(x)}{dx}\right)\left(\frac{dy_1(x)}{dx}\right)^{-1}\right]^2_{(x_S,y_S)}\,, \qquad (9)$$

where the two derivatives of geodesics are calculated in the coordinates of the source.

Let us now apply these general considerations to the case of a flat metric which is perturbed by the gravitational field of a moving MACHO. This means that the initial metric $g_{\mu\nu}^{(1)}$ is a Minkowski one while the perturbed metric $g_{\mu\nu}^{(2)}$ is the Schwarzschild one. Without MACHO, geodesics are straight lines emitted by the source, that is $r = r_0[\cos(\phi - \phi_0)]^{-1}$,

in polar coordinates, or $r_0 = Ax + By$, in Cartesian coordinates. The constants A and B are the same as above. When a MACHO (passing in the origin) perturbs the background, the geodesics are given by Eq.(5) (or (6)). By calculating the derivative in the position of the source (that is in $\{\pm a, 0\}$) and using (9), we get

$$\frac{\Delta L}{L} = \pm \left(\frac{R_s}{2r_0}\right)^2 \left\{\frac{A^2 + 2}{A\left[1 \pm A\left(\frac{R_s}{r_0}\right)\right]}\right\}^2 , \tag{10}$$

where plus sign means "focusing" and then a peak in light curve of the source detected by the observer, while minus sign means "defocusing" and then a gap in the light curve detected. Eq.(10) (by Eqs.(8)) shows that the variation of luminosity depends on the relative positions of the lens and the light ray (r_0, ϕ_0), on the mass of the MACHO M, as well as on the relative position of the lens and the light source ($\{\pm a, 0\}$).

Such calculations can be performed in any configuration of the system source–lens–observer. Here, for simplicity, we have taken into account source, lens and observer lying on the same line.

3   The mass of the lens and the optical depth

Using the above formulas, we can estimate the mass of of a MACHO acting as a lens both for focusing and defocusing cases. From Eq.(10), we have

$$M = \left(\frac{c^2 r_0}{G}\right) \left[\frac{A\sqrt{|\Delta L/L|}}{2 + A^2 \mp 2A^2\sqrt{|\Delta L/L|}}\right] , \tag{11}$$

where now minus sign refers to focusing and plus to defocusing. The modulus tells us that both the peak and the gap in light curve give indications on the MACHO mass.

By Eqs.(2) and (3), the optical depth is

$$\tau_\pm = \left(\frac{D_{ls}}{D_{ol}}\right)\left(\frac{r_0}{D_{ol}}\right) \left[\frac{A\sqrt{|\Delta L/L|}}{2 + A^2 \mp 2A^2\sqrt{|\Delta L/L|}}\right] , \tag{12}$$

where τ_+ is the optical depth (probability) connected to a focusing event while τ_- is associated to a defocusing one.

In order to give some numbers, we obtain a MACHO of mass $M \sim 0.5 \div 1 M_\odot$, if $\Delta L/L \sim 10^{-2}$, $r_0 \simeq r_E$ of the order of one astronomic unit and $\phi_0 \sim |\delta| + \pi/2$, with $|\delta| \sim 10^{-5}$. Such result holds for focusing and defocusing MACHOs. On the other hand, it is easy to obtain the optical depth $\tau \sim 10^{-6}$ toward the Galactic bulge and $\tau \sim 10^{-7} \div 10^{-8}$ toward the LMC [1],[11]. The similar results are also obtained for if $\Delta L/L \sim 10^{-4}$ and $|\delta| \sim 10^{-3}$. In principle, we can cover all the mass range $10^{-6} M_\odot$ to $10^2 M_\odot$ expected for MACHOs. However, we have to stress that, statistically, the features of the light curves are not expected to be completely symmetric: in fact, for

a randomly chosen focusing and defocusing configurations (I) and (II) in which the distances involved are the same, we can estimate that if the peak magnification is, for example, of a factor 3, the gap for defocusing is of the order 0.1.

4   Conclusions

We have pointed out that microlensing effects could be detected not only if we observe peaks in luminosity curve of sources, but also if we detect gaps. However, our prediction is that for observed to date quantity of events with given peak magnifications, must exist approximately the same quantity of events with gaps in light curve.

Furthermore, by the knowledge of the geometry (and the relative positions) of the optical system source–lens–observer, we can estimate both the mass and the optical depth for a given lens.

These facts could contribute to bypass one of the lack of microlensing detecting experiments: that is the low number of observed events (till now about 100, not all exactly tested, for millions of detected source stars). Roughly speaking, one could expect to double the number of succesful detections including also defocusing events.

It is worthwhile to note that when several MACHOs are present, the previous discussion still holds due to the Fermi principle (see, for example [10]). The effect is additive and it is similar to that of a light ray passing through different media with refraction indexes $n_1, \cdots n_j$. Then, in principle, it is possible to evaluate the total deviation of a light ray by summing up the effects of the various deflectors.

Finally, we have to stress that in a statistical approach to the microlensing, our approach gives rise to two contributions to the number density $n(D_l)$ of lenses, one coming from focusing objects $n_+(D_l)$ and another coming from defocusing ojects $n_-(D_l)$.

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